Technology Adoption by Firms and Distribution of Factor Income

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Abstract

This paper analyzes the impact of a decrease in the price of capital on the distribution of factor income through the heterogeneous impact across firms. I consider a directed search model with convex cost vacancy posting. In order to economize on vacancy costs, firms can either offer high wages to increase the filling rate or automate more to decrease labor demand. Theoretically, I show that highly productive firms automate more, and a decrease in the price of capital leads to higher automation. This leads to a lower labor share, a higher wage premium for non-routine workers, and higher residual wage dispersion. Quantitatively, the model implies that at the aggregate level, decrease in labor share is offset by the increase in capital share. Reallocation and firm-level decrease in labor share generates a seven p.p. decrease in aggregate. Unemployment risk generates inefficiency in the model, and with progressive taxation, the capital subsidy can increase the welfare of the new generation by 10%.

JEL classification: E23, J23, J3, O33.

Keywords: automation, labor share, technological adoption, firm heterogeneity, inequality.

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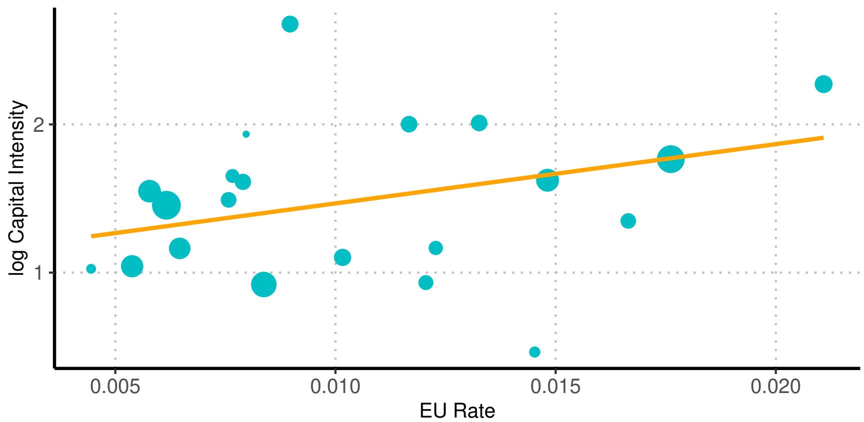
1 Introduction

Large firms use more advanced technology, have higher capital and skill intensity, and have lower labor share compared to the median firm (Zolas et al., 2021; Autor et al., 2020; Kehrig & Vincent, 2021). The gap has been increasing since the 1980s. The evidence suggests that improvement in technology benefits firms at the top of the productivity distribution more than firms at the median. In this paper, I analyze the impact of the decrease in the price of capital (or increase in capital productivity) on the distribution of factor incomes through the heterogeneous impact on firm-level technological choice.

To this end, I developed a model with directed search and endogenous technology adoption by firms. Firms must complete a set of tasks to produce the final good, similar to the task-based framework analyzed in Acemoglu & Restrepo (2018). Each task can be produced with capital or labor, and firms endogenously assign their capital and routine labor across tasks. Under the assumption of a perfectly competitive capital and labor market and homogeneous production function (for example, CES), a firm's capital intensity does not depend on the firm's productivity (Sato, 1977). Homogeneity implies that the marginal rate of technical substitution depends on capital intensity (capital over labor), and perfectly competitive labor and capital markets imply that every firm faces the same relative prices. Hence all firms should choose the same capital per labor. To break this result, I consider convex adjustment cost through search friction, as in Kaas & Kircher (2015). To economize on vacancy costs, a firm has two options. First, it can offer a higher wage to increase the job-filling rate and, hence post fewer vacancies. Second, it can automate more tasks to decrease the labor demand.

Large firms need to replace more labor due to exogenous labor turnover than small firms. Theoretically, I show that large firms utilize both channels more intensively: they offer higher wages and automate more. Figure 1 shows that the industries that experience higher job separation (measured as the monthly transition rate from employment to unemployment (EU rate), calculated from monthly IPUMS CPS (Flood et al., 2018)) have higher capital

Figure 1: Labor Turnover and Capital Intensity



Note: EU rate is the average monthly transition from employment to unemployment, calculated from monthly IPUMS CPS (Flood et al., 2018). Capital intensity is the ratio of equipment capital to employment, calculated from WRDS Compustat. Each point represents an industry, weighted by employment size. Data is for 2017-2018.

intensity. Industry-level capital intensity is defined as equipment net property, plant, and equipment divided by employment from Compustat accessed via WRDS. The data is for 2017-2018. A percentage point increase in the EU rate leads to a 40% increase in capital intensity. The positive relation suggests that firms' optimal choice of production technology depends on the labor turnover rate.

Acemoglu & Shimer (1999) show that directed search might be inefficient when individuals are risk averse. This result extends to this model also. When individuals are risk averse because they cannot insure themselves against unemployment risk, they inefficiently search for jobs with a high probability of match but pays low wages. This has two implications. First, wages offered in the equilibrium will be lower than the socially optimal level. Therefore, small firms use inefficiently high labor-intensive technologies. Second, the wage schedule is steeper in the market equilibrium compared to an efficient wage schedule. In order to compensate for lower job finding probability, a firm needs to increase wages more than the social planner. For this reason, large firms utilize automation technology at a higher rate to decrease their labor demand.

As a result, the capital intensity aggregate level might be too low if the former channel dominates or too high if the latter channel dominates. Golosov et al. (2013) show that if firms are single labor firms, then regressive taxation recovers the efficient allocation. Because the primary source of inefficiency is the same, it is also true for this model. However, suppose regressive taxation is not possible due to a lack of public support. In that case, the inefficiency in the technology choice creates room for the government to subsidize or tax capital usage. The sign of capital tax depends on which channel dominates.

In the quantitative part, I calibrate the model to the 1980s US economy. Top 10% productive firms mainly drive the aggregate level statistics. The labor share is above the aggregate level for the bottom 80%, but it steeply decreases among top firms. Similarly, the dispersion of technology outside the top is small, with a 20% difference between the 90th percentile and 10th percentile. In contrast, the difference is that a firm in the 99th percentile automates two times more than the 90th percentile. However, the wage dispersion generated by the model is not high. The wage difference between the top 10% and bottom 10% is around 4%. This is not surprising given that search friction fails to generate high wage dispersion (Hornstein et al., 2011), and it generates half of the observed dispersion between production workers in high and low technology-intensive firms (Doms et al., 1997).

In the model, a capital price reduction impacts the aggregate economy through two channels. First, it reallocates labor and capital to more productive firms. As the price of capital decreases, the wage rate for non-routine labor increases since they complement capital. This leads the lowest productive firms to exit the market, increasing average firm productivity in the economy. Second, at a given firm productivity, the capital intensity and non-routine intensity increase. This leads to a decrease in labor income share and a rise in non-routine wage premiums. 45% reduction in the price of capital, similar to the realized decrease in the relative price of an investment, lowers the labor share by seven percentage points (p.p.). At the firm level, all firms' labor share declines. Outside the top 10% of firms, all the reduction is offset by an increase in capital share, whereas for top firms, there is also a reduction in profit share by 2 p.p.. The main reason for the decrease in profit share is that the rental rate is equal to marginal product of capital. In contrast, the wage rate is lower than marginal product of labor. However, the level of profit is increasing thanks to scale effect. The size of the decline in the price of capital has a heterogeneous impact. The gap between the top 10% firms and the rest of the distribution increases with the size of the decline.

Lastly, I quantitatively analyze the optimal capital taxation under the progressive income tax regime. The model implies that a 15% tax subsidy maximizes the welfare of newborn unemployed workers for the calibrated tax schedule. In other words, in the economy, small firms are not utilizing automation technology enough. This provides a rationale for the lower capital tax rate compared to the labor income tax rate, as documented by Acemoglu et al. (2020).

Related Literature: There is extensive literature on the aggregate decline in labor share. There are a few possible explanations discussed in the literature. A set of papers discussed the impact of technological development on labor share through capital-labor substitution (see, for example, Karabarbounis & Neiman (2014)). Another strand of papers discussed measurement error due to the imputation of self-employment income (Elsby et al., 2013) or shifting labor income to business income (Smith et al., 2019). Another explanation is the reallocation of economic activity to large firms, which tend to have lower labor share Autor et al. (2020); Kehrig & Vincent (2021). Barkai (2020) and Gutiérrez & Piton (2020) claim that the decrease in labor share is offset by an increase in the profit share. This paper models the endogenous technology choice of firms and quantitatively analyze how changes in technology generate aggregate effect through reallocation and technology adaption by firms.

Hubmer & Restrepo (2021) and Firooz et al. (2022) also analyze the heterogeneity of technology adoption across firms. The main mechanism they consider in these papers is the fixed cost of technology adoption. Because of the fixed cost, low-productive firms do not adopt new technologies after a price decrease, which creates a dispersion in labor shares across firms. I consider convex labor adjustment cost as the reason for the dispersion in the labor share.

The model is based on the literature that studies the impact of search friction on firm size. Elsby & Michaels (2013) analyze firm size distribution within a random search environment, Kaas & Kircher (2015) study the firm dynamics with directed search friction, and Bilal et al. (2022) investigate random search with on-the-job-search. Leduc & Liu (2022) analyze the automation choice over the business cycle with search friction to explain the volatility in unemployment. This paper utilize directed search friction with endogenous technology adoption. I use a task-based framework, as in Acemoglu & Restrepo (2018), and analyze how costly labor adjustment impacts the automation choice of firms.

This paper also contributes to the burgeoning literature on the impact of automation on income inequality. Impact of automation on employment and wages extensively shown (Acemoglu & Restrepo, 2021, 2020; Aghion et al., 2020). By replacing middle-skilled routine workers and complementing non-routine workers, automation leads to wage polarization and non-routine wage premium (David & Dorn, 2013; Jaimovich & Siu, 2020). Leonardi (2007) shows that residual wage inequality is increasing with the capital intensity at the industry level. Koru (2019) provides a theory that link automation to the Pareto parameter of top income distribution and Koru (2020) shows that automation can explain one fourth of the rise in wealth concentration. Acemoglu & Shimer (2000) provide a model that jointly explains technology and wage dispersion among identical firms. I contribute to this literature by analyzing automation's aggregate and firm-level impact with convex labor adjustment cost.

The structure of the paper is as follows: section 2 describes the static model and provides the main channels of the model. Section 3 describes the dynamic model with two types of labor. Section 4 provides the quantitative analysis. Section 5 discusses optimal capital taxation. Section 6 concludes.

2 Static Model with One Type Labor

In this section, I analyze a static model with one type of labor to provide insight into the main trade-off in the model. In the section 4, I consider a dynamic model with two types of labor.

2.1 Labor Market

Labor market characterized by directed search, similar to Kaas & Kircher (2015). Firms post long-term fixed-wage contracts, and workers direct their search toward the most attractive contract. In any contract market, firms and workers match through constant returns to scale matching function, which implies that the relevant parameter for a market is the vacancy-to-unemployed ratio or market tightness. Let θ denotes the market tightness. Furthermore, let $p(\theta)$ be the probability that a worker matches with a firm and $q(\theta)$ be the probability that a vacancy matches with a worker. Underlying matching function implies that $\theta q(\theta) = p(\theta)$. I assume that matching function satisfies $q'(\theta) < 0$ and $p'(\theta) > 0$. In other words, as θ increases, the market becomes congested for firms; hence it becomes harder for a firm to match with a worker. On the other hand, since there are more vacancies per labor, it becomes easier for a worker to find a job.

In equilibrium, individuals must be indifferent to searching for any contracts offered. This implies that there should be a one-to-one relationship between the wage offered and the market tightness. Hence, each submarket defined by market tightness, and an associated wage rate $w(\theta)$ makes individuals indifferent to any other active submarket.

2.2 Preferences

There is a unit measure of labor. Each labor has the same efficiency in the labor market and supplies one unit of labor inelastically. They are risk averse, and their utility is given by $u(c)$, where c is the consumption and equals to wage rate if they are employed or unemployment benefit b if they are unemployed.

At the beginning of the model, everyone is unemployed, and they choose which market to search for a job in. Upon choosing the market, with probability $p(\theta)$ they find a job and their utility is $u(w(\theta))$. With $(1-p(\theta))$, they cannot find a job and their utility is $u(b)$ in this case. The problem of an individual is given by:

$$\max_{\theta} p(\theta)\left[u(w(\theta)) - u(b)\right].$$

2.3 Technology

Technology is similar to Acemoglu & Restrepo (2018). There is a unique final good, and firms use labor and capital to produce the final good. Firms access the same production technology, but they differ in terms of total factor productivity, which I denote by z. Suppose z is drawn from some distribution function $G(z)$ $[\underline{z}, \bar{z}] \subset \mathbb{R}$, with possibility that $\bar{z} = \infty$. Let $g(z)$ be the pdf of G.

The production function is similar to Acemoglu & Restrepo (2018). To produce a unit of the final good, a firm must complete a unit interval measure of tasks. The production function of task $i \in [0,1]$ is given by:

$$y_i = \psi_i k_i + \gamma_i \ell_i,$$

where ψ_i and γ_i are the capital and labor productivity, respectively. What matters for the optimal choice is the relative productivity, γ_i/ψ_i. Assume that these tasks are ordered so that labor has a comparative advantage in high-index tasks. In other words, the relative productivity is increasing with i.

Assumption 1. $\dfrac{\gamma_i}{\psi_i}$ *is increasing in* i.

The final good is produced with a CES aggregator, with decreasing returns to scale function:

$$Y(\{y_i\}_i; z) = z\left[\int_0^1 y_i^{\rho}\right]^{\alpha/\rho}$$

where $\alpha \in (0,1)$ controls the decreasing returns to scale, $\rho < 1$ is the substitution parameter, and z is the firm productivity. In other words, capital and labor are perfect substitutes within a task, but across tasks, they are imperfect substitutes (and possibly they are complements).

A firm with productivity z chooses which market to search for workers, how many workers to hire, how much capital to rent, and how to allocate capital and labor across tasks.

$$\pi(z) = \max_{\{\ell_i, k_i\}, \theta} z \left[\int_0^1 (\psi_i k_i + \gamma \ell_i)^\rho \, di \right]^{\frac{\alpha}{\rho}} - w(\theta) \int_0^1 \ell_i - v\left(\frac{\int_0^1 \ell_i di}{q(\theta)} \right) - r \int_0^1 k_i di, \qquad \text{(FP)}$$

$$k_i \geq 0, \ell_i \geq 0,$$

where $v(.)$ is the vacancy cost function, $q(\theta)$ is the rate at which the firm meets with a worker. $\int \ell / q(\theta)$ is the total vacancy posted to hire $\int \ell$ labor. r is the cost of capital. Following Kaas & Kircher (2015), I assume that v is convex. Due to search friction, firms have two margins to attract the desired level of labor: posting more vacancies and offering higher wages. Due to the convexity of the cost function, the optimal mix of these two channels depends on the firm's characteristics, as shown in the data by Davis et al. (2013). The empirical evidence for such convex cost is provided by Blatter et al. (2012) and Merz & Yashiv (2007).

There is a large mass of firms that decide to enter the market. The cost of entry is $\kappa > 0$. Upon entering the market, firms draw the productivity z from a distribution $G(z)$.

Definition 1. *An equilibrium is a tuple of value of searching for a job S, mass of firms M, labor demand and capital demand of firms $\ell_i(z), k_i(z)$, market choice of firms $\theta(z)$, wage function $w(\theta)$ that satisfies:*

- *Individuals are indifferent across markets:*

$$\max_\theta p(\theta) \left[u(w(\theta)) - u(b) \right] \leq S$$

for any θ, and holds with equality if there exist z such that $\theta = \theta(z)$ (i.e. market is active).

- $\ell_i(z), k_i(z), \theta(z)$ *solve firm's problem* (FP).
- *Firms profit is zero ex-ante due to free entry condition:*

$$\int \pi(z)dG(z) = \kappa.$$

- *Labor market clears, i.e., the total number of job searchers is equal to 1:*

$$M \cdot \int \frac{\int \ell_i(z)di}{p(\theta(z))} dG(z) = 1.$$

The first condition implies that individuals are indifferent between offered contracts, and the return to search S is the same in any active market θ. The second condition is the optimality condition for firms. The third condition is the free entry condition. In the equilibrium, if there is positive profit, more firms enter the market. If it is negative, firms exit the market. In equilibrium, the expected profit must be equal to the entry cost. The fourth condition is the labor market clearing condition. If $u(\theta)$ workers search for a job in market θ, then total employment is $p(\theta(z))u(\theta(z)) = M \int \ell_i dig(z)$. Total job searchers are the some of all $u(\theta)$, which must be equal to 1.

2.4 Characterization of the equilibrium

In this subsection, I characterize the equilibrium. First, I show that the wage rate is decreasing with θ. Then, I simplify the solution to the firm's problem by reducing it to a system of equations with two unknowns and two equations. Then, I show how firm productivity affects this system of equations and how technology choice changes across firms.

2.4.1 Equilibrium Wage Rate

The first equilibrium condition states that individuals must be indifferent between markets. This implies that for a given search value S, one can figure out the wage rate as:

$$w(\theta) = u^{-1}\left[\frac{S}{p(\theta)} + u(b)\right]. \tag{1}$$

Derivative with respect to θ is:

$$w'(\theta) = -\frac{p'(\theta)}{p(\theta)^2}\frac{S}{u'(w(\theta))}.$$

Because $p(.)$ and $u(.)$ are increasing, $w(\theta)$ is decreasing with θ. This is intuitive. As θ decreases, a worker's probability of finding a job decreases, leading to higher unemployment risk. This implies that the wage offer needs to compensate for this additional risk, and, hence, the wage rate should rise as market tightness decreases. The following lemma characterizes the behavior of the wage function as $\theta \to \infty$, which will be useful later to characterize the behavior of large firms.

Lemma 1. *As $\theta \to \infty$:*

- $w(\theta) \to u^{-1}\left[S + u(b)\right] < \infty.$
- $w'(\theta) \to 0.$

All proofs are in the appendix.

2.4.2 Solution to the Firm's Problem

In this section, I consider the problem of a firm defined in (FP). A firm chooses labor and capital for each task and where to search for a worker.

Let's start with the optimal choice of θ. Consider a firm that wants to hire L workers, so it posts $L/q(\theta)$ vacancies. It can choose high θ (hence lower wage) and post high enough vacancies to match with L workers. Or it can decrease θ (hence offer lower high wage) but economize on vacancy posting. The optimal choice implies that the marginal benefit of increasing θ (lower wage) must be equal to the cost of it (higher vacancy posting), formally:

$$v'\left(\frac{L}{q(\theta)}\right)\frac{q'(\theta)}{q(\theta)^2} = w'(\theta). \tag{2}$$

The critical assumption here is that v is convex. This creates the main trade-off in the model. Convexity leads to the optimal decision to be a function of the productivity of the firm. If v is linear, then v' is a constant; hence the optimal choice of θ does not depend on employment size, therefore, the firm productivity.

Now consider the optimal technological choice for task i. Because labor and capital are perfect substitutes within a task i, only one of them is going to be used to produce task i. A firm chooses the input that has a lower effective marginal cost (i.e., marginal cost over productivity). Marginal costs do not depend on automation choice, but the capital and labor productivity are increasing in I. Assumption 1 implies that the productivity of capital increases at a slower rate than labor productivity. This implies that there is a cutoff I such that for $i \leq I$, the firm uses capital, and for $i > I$ firm uses labor, and at I it is indifferent, i.e.:

$$\frac{w(\theta) + v'\left(\frac{L}{q(\theta)}\right)\frac{1}{q(\theta)}}{\gamma_I} = \frac{r}{\psi_I}. \tag{3}$$

where the left-hand side is the effective marginal cost of labor and the right-hand side is the effective marginal cost of capital.

First-order conditions with respect to capital and labor are:

$$z\alpha\left[\int_0^1 (\psi_i k_i + \gamma\ell_i)^\rho \, di\right]^{\frac{\alpha}{\rho}-1} \gamma_i^\rho \ell_i^{\rho-1} = w(\theta) + v'\left(\frac{\int \ell}{q(\theta)}\right)\frac{1}{q(\theta)} \qquad \text{for } i > I,$$

$$z\alpha\left[\int_0^1 (\psi_i k_i + \gamma\ell_i)^\rho \, di\right]^{\frac{\alpha}{\rho}-1} \psi_i^\rho k_i^{\rho-1} = r \qquad \text{for } i \leq I.$$

This conditions implies that $\gamma_i^\rho \ell_i^{\rho-1} = \gamma_I^\rho \ell_I^{\rho-1}$ for $i > I$ and $\psi_i^\rho k_i^{\rho-1} = \psi_I^\rho k_I^{\rho-1}$ for $i \leq I$. Furthermore, together with equation (3) implies that $\gamma_I \ell_I = \psi_I k_I$. In words, because the firm is indifferent to automating the marginal task (effective marginal costs are the same), the production level is the same whether it uses labor or capital. Using these conditions, total demand for capital ($K = \int k_i di$) is given by:

$$K = L\frac{\phi(I)\gamma_I}{\eta(I)\psi_I}, \tag{4}$$

where

$$\phi(I) = \int_0^I \left(\frac{\psi_i}{\psi_I}\right)^{\frac{\rho}{1-\rho}}, \quad \eta(I) = \int_I^1 \left(\frac{\gamma_i}{\gamma_I}\right)^{\frac{\rho}{1-\rho}}.$$

Observe that at the optimal choice, the marginal cost of labor becomes $r\gamma_I/\psi_I$ thanks to condition (3). Imposing $\gamma_I \ell_I = \psi_I k_I$ to first-order condition for ℓ_I leads to:

$$z\alpha \left[\phi(I) + \eta(I)\right]^{\frac{\alpha}{\rho}-1} \gamma_I^\alpha \left(\frac{L}{\eta(I)}\right)^{\alpha-1} = \frac{r\gamma_I}{\psi_I}. \tag{5}$$

The solution to the firm's problem is optimal choice of $\{K, L, I, \theta\}$ that satisfies conditions (2), (3), (4), and (5). We can reduce this system of equations to two equations in automation level I and market tightness θ. First, observe that total capital demand does not impact conditions (2), (3), or (5). Hence, condition (4) gives the optimal capital demand as a function of L and I. Combining (2) and (3) provides the first equation that links I to θ:

$$\frac{r\gamma_I}{\psi_I} = w(\theta) + w'(\theta)\frac{q(\theta)}{q'(\theta)}. \tag{6}$$

Let $\theta_1(I)$ be the solution of this function given some I.

Now consider conditions (2) and (5). Observe that marginal (5) does not depend on market tightness because the marginal product of labor does not depend on θ, and automation is always adjusted so that the marginal cost of labor equals the right-hand side. This provides one to one relationship between automation level I and labor demand L. Imposing this to condition (2) provides the second equation that links automation to market tightness. Let define $\theta_2(I)$ be the solution to (2) and (5) given some I.

This reduces the problem of a firm to two equations. $\theta_1(I)$ satisfies condition (6), $\theta_2(I)$ satisfies conditions (2) and (5). The intersection of these two functions provides the optimal I and θ. Given I, (5) pins down L and (4) pins down K. To prove that a solution to this problem exist, I need to show $\theta_1(I)$ and $\theta_2(I)$ intersect at some $I \in [0, 1]$. To show that, I first prove $\theta_1(I)$ is a decreasing function and $\theta_2(I)$ is an increasing function. Then, I show that $\theta_2(I) > \theta_1(I)$ as $I \to 1$ and $\theta_2(I) < \theta_1(I)$ as $I \to 0$. By the intermediate value theorem, these conditions satisfy that they intersect.

The following assumption makes sure $\theta_1(I)$ is a decreasing function.

Assumption 2. $\zeta(\theta) \equiv -\frac{p'(\theta)q(\theta)}{p(\theta)q'(\theta)}$ *is nonincreasing in* θ.

This assumption is satisfied in most of the common matching functions[1]. Under this assumption, $w'(\theta)q(\theta)/q'(\theta)$ is a nonincreasing function.

Lemma 2. $\theta_1(I)$ *is decreasing in* I.

The intuition is simple. If a firm uses both capital and labor, then the optimal choice

[1]For example, it is constant if a matching function is Cobb-Douglas.

implies that the effective cost of labor and capital must be the same at the marginal task (equation (6)). As the automation level increases, this increases the relative productivity of labor at the marginal task (by assumption 2), which makes labor cheaper. Hence, the firm can choose lower θ, which increases the marginal cost of labor. In another way, an increase in θ decreases the marginal cost of labor and makes it cheaper compared to capital. Hence, the firm now wants to automate less.

There are two important observations. First, $\theta_1(I)$ does not depend on z. This is because the relative productivity of labor at the marginal task does not depend on z. Second, right hand side does not converge to 0 as $\theta \to \infty$, by lemma 1. Hence, there is a lower bound on the equilibrium values of I, if $r\gamma_0/\psi_0$ is lower than the limit of the wage. This will be important to prove the existence of a solution.

Now, I show that this is an increasing function.

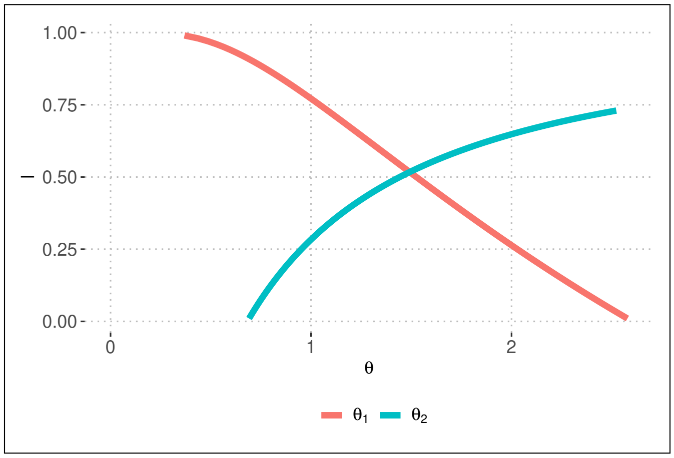
Lemma 3. *$\theta_2(I)$ is increasing in I.*

This result is intuitive. As a firm automates more tasks, labor demand decreases (by equation (5)). Recall that a firm that wants to hire L workers has two margins to adjust: vacancy posting and wage (by equation (2)). As L decreases, the firm posts lower vacancies and adjusts its wage downward. This implies that it chooses higher θ. Therefore θ_2 is increasing with I.

Proposition 1. *There exists a unique solution to the firm's problem. In other words, there exists $I^\star$ such that $\theta_1(I^\star) = \theta_2(I^\star)$. Furthermore, if γ_i/ψ_i converges to 0 as $i \to 0$, and it diverges as $i \to 1$, the solution is interior, $I^\star \in (0,1)$.*

Figure 2 summarizes the solution to the firm's problem. There are two functions that are strictly increasing and strictly decreasing. By the intermediate value theorem, they must intersect if their endpoints are on the opposite side. If not, then the firm's automation decision is a corner solution. However, if I assume that the relative productivity of labor goes to zero as the index of the task goes to 0, then $\theta_2(I) < \theta_1(I)$ for I is small enough. Furthermore, if I assume that relative productivity diverges as I goes to 1, the $\theta_1(I) < \theta_2(I)$

Figure 2: Solution to the Firm's Problem



for large I, which implies that they must intersect at an interior point.

The following proposition shows that the optimal choice of I is increasing with z, i.e., higher productive firms automate more tasks.

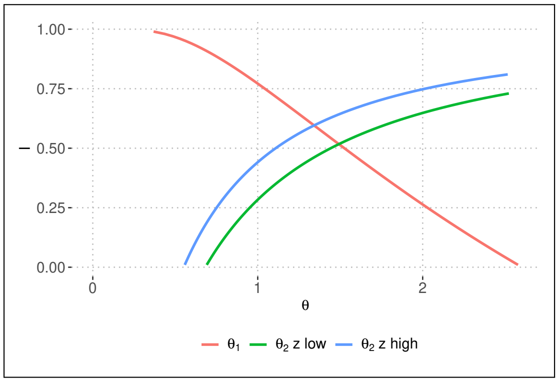
Proposition 2. *The optimal automation decision $I^{\star}(z)$ is increasing in z.*

This is straightforward to see. Recall that z does not impact $\theta_1(I)$. Now consider θ_2. Fix I, and see what happens to θ_2 as z increases. An increase in z increases labor demand because a firm is more productive, it wants to hire more labor. This implies that it needs to post more vacancies. So, the firm will decrease θ to increase worker finding probability so that it can economize on vacancy posting. Mathematically, z increases $L(I, z)$, which leads the left-hand side of (5) to go up. Hence, θ should go down to bring that equation to equality. In other words, an increase in z shifts θ_2 to the left, and now it will intersect with θ_1 at a higher I.

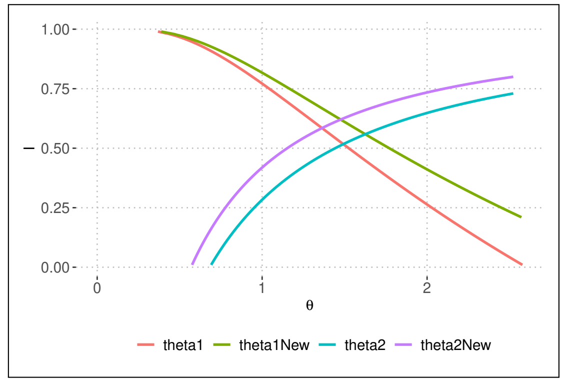
Proposition 3. *In the partial equilibrium (fixing search value fixed), the optimal automation decision $I^{\star}(z)$ is decreasing in r.*

A decrease in r is more complicated because it impacts both functions. On the one hand, it makes capital cheaper. To equate the relative effective marginal productivity, a firm wants to decrease wage (increase θ). Thus, $\theta_1(I)$ shifts to the right. On the other hand, because capital is cheaper, at the same automaiton level, firm wants to hire more workers.

Figure 3: Comparative Statistics



(a) Increase in z



(b) Decrease in r

As L goes up, the firm chooses a higher wage (lower θ) to economize on vacancy posting. Hence $\theta_2(I)$ shifts to left. This implies that automation decisions will increase for all z. However, the impact on θ is ambiguous. It depends on which effect to dominate. Figure 3b illustrates this impact. Capital per labor increases because from the fourth optimality condition $K/L = \frac{\phi(I)\psi_I}{\eta(I)\gamma_I}$, which is increasing in I. However, the general impact depends on how S changes because it will impact the wage function.

To summarize, in the model, highly productive firms have more workers, automate more, and they use more capital per labor compared to lowly productive firms. As the price of capital decreases, top firms automate more and use more capital per labor, and the gap increases. However, the overall impact on wage and employment is unclear because there is an ambiguous impact on the choice of θ.

2.5 Inefficiency of the Market Equilibrium

In this section, I consider the social planner's problem. The first best allocation equates to the marginal utility of individuals independent of their employment status, i.e., the consumption level of employed and unemployed are the same. But this implies that the return to search is 0. Therefore it cannot be supported in the market equilibrium.

Therefore I focus on the second best allocation in which the social planner needs to keep search value as in the market equilibrium. For each firm z, she chooses in which market to search for job ($\theta(z)$), how many labor to hire, $\ell_i(z)$, how many capital to rent $k_i(z)$ for task i. Conditional on finding a job at firm z, she chooses consumption level $c(z)$, and a common consumption level for unemployed c_u. Lastly, she decides how many firms to establish, M. Because in the market equilibrium, the social planner cannot distinguish the search behavior of individuals, it chooses a uniform consumption level for unemployed individuals, irrespective of where they searched for a job.

Because the search value is fixed, the social planner wants to maximize the utility of the unemployed. In other words, for a given search value S, $p(\theta(z))u(c_z)+(1-p(\theta(z))u(c_u) = S + u(c_u)$. Formally, the social planner solves the following problem:

$$
\begin{aligned}
\max \quad & u(c_u) \\
s.t. & \int_z c_z \left(\int_i \ell_i(z)di\right) MdG(z) + (1 - \int_z \int_i \ell_i(z)diMdG(z))c_u \leq \\
& \quad \int_z \left[z \left[\int_0^1 (\psi_i k_i + \gamma \ell_i)^\rho \, di \right]^{\frac{\alpha}{\rho}} - v\left(\frac{\int_0^1 \ell_i di}{q(\theta)}\right) - r\int_0^1 k_i(z)di \right] MdG(z) + \\
& \quad (1 - \int_z \int_i \ell_i(z)MdG(z))b - M\kappa, \\
& \int_z \frac{\int_i \ell_i(z)di}{p(\theta(z)} MdG(z) \leq 1, \\
& p(\theta(z))u(c_z) + (1 - p(\theta(z))u(c_u) \leq S.
\end{aligned}
$$

The first constraint is the resource constraint. Total consumption must be less than the production. The second constraint is the labor market clearing condition. $\frac{\int_i \ell_i(z)di}{p(\theta(z)}$ job seekers needed at market $\theta(z)$ to achieve desired employment. Total job seekers must be less than 1. The third constraint is the search value constraint.

This problem is very similar to Golosov et al. (2013). They consider single vacancy firms; here, I assume that firms are large and decide their production technology. However, the solution is very similar. Under the assumption that u is concave, the market equilibrium is not efficient, but it can be recovered with regressive taxation.

Proposition 4. *The market equilibrium is inefficient. However, it can be recovered with the following tax schedule:*

$$1 - \tau(\theta) = \frac{c_{z(\theta)}}{c_{z(\theta)} - c_u + b + \lambda/p(\theta)}.$$

where λ is the ratio of the Lagrange multiplier of resource constraint to the Lagrange multiplier of labor market clearing condition and $z(\theta)$ is the firm productivity that chooses θ.

Even though the directed search is usually efficient, that depends on the utility function. Acemoglu & Shimer (1999) show that when workers are risk averse, the directed search is not efficient because of the uninsured unemployment risk. Workers are not taking a risk when they search for a job; hence they choose higher θ, which increases their job finding probability. However, the social planner reallocates some workers to lower θ, which increases production because it increases the matching rate for firms. As production increases, the social planner reallocates some of these resources to unemployed workers to compensate for their risk. However, without an insurance market, workers do not take that risk. This is also true for this model. Hence, under the assumption that workers are risk averse, the market will not be efficient.

The natural question here is how this inefficiency is reflected in technological choice. There are two impacts. First, because workers are not taking enough risk, the wage rate will be lower than the efficient level. Hence, this incentivizes firms to use more labor and automate less than socially optimal levels. On the other hand, it will become harder for large firms to find workers. They would like to offer higher wages to attract more workers and economize on vacancy postings. But this might not be possible in market equilibrium because low θ markets are not active. This provides an incentive for large firms to automate more compared to the socially optimal level.

3 Dynamic Model with Two Types of Workers

In this section, I extend the previous model to a dynamic version with two types of labor: routine and non-routine. Routine workers are subject to capital substitution, whereas non-routine workers are not. This way, I can analyze the impact of change in the price of capital on inequality among similar workers and between different occupations. I assume that the labor market is segmented by occupation, but they have the same matching function. Time is discrete.

3.1 Preferences

Let μ_N and μ_R be the measure of routine and non-routine workers. Workers discount future with $\beta \in (0, 1)$, and with probability $(1 - \sigma)$ they die. Let U_i and $V_i(w)$ be the lifetime value of unemployed and employed at wage rate w for $i \in \{N, R\}$.

Capital accumulation impacts the job search behavior of individuals, and it creates precautionary search behavior (Eeckhout & Sepahsalari, 2021; Chaumont & Shi, 2022). Therefore I make two further assumptions. First, I assume that routine workers cannot save or borrow. In other words, they are hand-to-mouth consumers. Only non-routine workers can save. Second, I assume that routine workers are risk averse and non-routine workers are risk neutral. This way, wealth level does not impact the job search behavior of non-routine workers. However, the risk aversion of routine workers leads to inefficiency in the market and potential room for the government.

Formally, the value functions of routine workers are as follows:

$$U_R = u(b) + \beta\sigma \left[\max_{\theta} p(\theta)(V(w(\theta)) - U) + U\right],$$

$$V_R(w) = u(T(w)) + \beta\sigma \left[\delta U + (1 - \delta)V(w)\right],$$

Where $T(w)$ is the after-tax income. Similarly, value functions of non-routine workers with asset level a are:

$$\begin{aligned}
U_N(a) = \max_{a'} b + r(1-\tau_k)a + q(1-\delta_k)a - qa' + d + \\
\beta\sigma \left[\max_{\theta} p(\theta)(V_N(w(\theta), a') - U_N(a') + U_N(a')\right], \\
V_N(w, a) = \max_{a'} T(w) + r(1-\tau_k)a + q(1-\delta_k)a - qa' + d + \\
\beta\sigma \left[\delta U_N(a') + (1-\delta)V_N(w, a')\right],
\end{aligned}$$

Where r is the rental rate, q is the price of capital, δ_k is the depreciation rate, and d is the dividend payments.

I focus on progressive taxation analyzed in Heathcote et al. (2017). Even though regressive taxation is optimal in this model (Golosov et al., 2013), due to public support for progressive taxation (Ballard-Rosa et al., 2017), it is not politically feasible. This exacerbates the inefficiency in the market equilibrium.

3.2 Technology

Production technology is similar to the static version. But now final good is produced by:

$$zN^{1-\alpha}\left[\int_0^1 y(i)^\rho di\right]^{\frac{\alpha}{\rho}},$$

where N is the measure of non-routine workers employed by the firm. Firms have the same discount factor and maximize future profits' present value. Observe that present value of wage payment to a worker is $w/(1-\beta\sigma(1-\delta))$. Hence, a firm's lifetime value is linear in the wage offered. Then, the firm's value function is

$$\begin{aligned}
F(z, R, N) = \max_{\theta_N, \theta_R,, N', R'} \pi(R', N', z) - \frac{w_R(\theta_R)(R' - \sigma(1-\delta)R)}{1-\beta\sigma(1-\delta)} - \frac{w_N(\theta_N)(N' - \sigma(1-\delta)N)}{1-\beta\sigma(1-\delta)} \\
- v\left(\frac{(R' - \sigma(1-\delta)R)}{q(\theta_R)}\right) - v\left(\frac{(N' - \sigma(1-\delta)N)}{q(\theta_N)}\right) + \beta F(z, R', N'),
\end{aligned}$$

where π is defined as:

$$\pi(R, N, z) = zN^{1-\alpha} \left[\int_0^1 (\psi_i k_i + \gamma_i \ell_i)^\rho di \right]^{\frac{\alpha}{\rho}} - r \int_0^1 k_i di$$
$$s.t. \quad \int_0^1 \ell_i di \leq R.$$

3.3 Government Budget

Government has exogenous spending, GS. We assume this is a waste and financed period by period by income, capital income, and profit taxation.

3.4 Definition of an Equilibrium

For given price of capital q, tax rates $T(.), \tau_K, \tau_\pi$, the stationary equilibrium consists of wage function $w_i\theta$, rental rate r, automation decision $I(z)$, labor demand $N(z), R(z)$, capital demand $K(z)$, market choice $\theta(z)$, value functions U_i, V_i, F, search values S_i for $i \in \{N, R\}$ such that:

- Value functions satisfy Bellman equations.
- Labor demand N, R, automation decision I, and θ_i solve the firm's problem.
- Individuals are indifferent to search at any active market:

  $$p(\theta)\left[V_i(w_i(\theta)) - U_i\right] \leq S_i,$$

  holds with equality if there exists z such that $\theta(z) = \theta$.

- Labor market clears:

$$\int \frac{(1-\sigma(1-\delta))R(z)}{p(\theta(z))} dG(z) = \sigma(1 - \int R(z) dG(z)) + (1-\sigma)\mu_R,$$
$$\int \frac{(1-\sigma(1-\delta))N(z)}{p(\theta(z))} dG(z) = \sigma(1 - \int N(z) dG(z)) + (1-\sigma)\mu_N,$$

- The rental rate satisfies the following:

$$r = \frac{q}{1-\tau_K}\left[\frac{1}{\beta\sigma} - 1 + \delta_k\right].$$

- Government budget balanced:

$$\int [(w_r(z) - T(w_R(z)))R(z) + (w_N(z) - T(w_N(z)))N(z) + \tau_K K(z) r + \tau_\pi \pi(L(z), N(z), z)]\, dG(z) = GS.$$

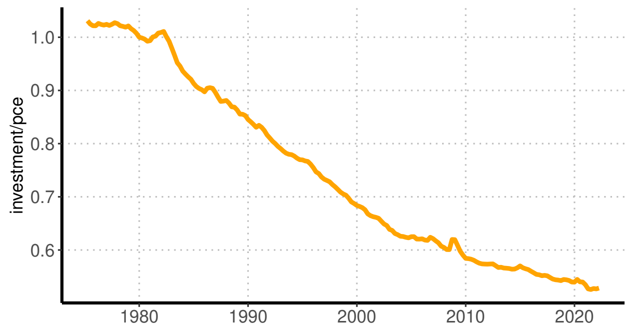
4 Quantitative Analysis

In this section, I quantitatively analyze the impact of a decrease in the price of capital. To do this, I calibrate my model to the 1980s US economy and then simulate my model after a decrease in capital price. Figure 4 shows the change in the price of investment to the price of consumption goods. I use BLS's price deflator for investment to price deflator for personal consumption expenditure as the measure and normalized it to 1980. As can be seen, the price of capital was stable from the 1970s to the mid-1980s. Afterward, there is a sharp decrease, with a 45% decrease in 40 years. I choose 1980 as the initial steady state and then consider the impact of a 45% decrease in the prices in the new steady state.

4.1 Calibration

The period in the model is a year because of the progressive tax schedule. I set the time discount factor $\beta = 0.96$ as it is standard in the literature, and survival rate $\sigma = 0.98$, which

Figure 4: Change in the Relative Price of Investment



implies 50 years of life expectancy after entering the labor market. Following Guerreiro et al. (2021), I set $\mu_N = 0.4$,and $\mu_L = 0.6$.

I assume that utility of routine workers is $u(c) = log(c)$, and utility of non-routine workers is $u(c) = c$. The unemployment benefit is chosen to be 50% replacement ratio, which is the case for most states in the US (Department of Labor, 2021).

I assume that productivity is distributed by a Pareto distribution with shape parameter λ_z and scale parameter $\underline{z}$. Notice that number of firm M and $\underline{z}$ cannot be separately identified. This is because the conditional distribution of a Pareto distribution is also a Pareto distribution. Formally, what matters in the model is $Mg(z) = M\lambda_z \underline{z}_z^{\lambda}/z^{\lambda_z+1}$, and relevant parameter is $(M^{1/\lambda_z}\underline{z})^{\lambda_z}$. So I fixed M and calibrated $\underline{z}$. These are calibrated to match the employment share of the top 1% of firms and employment level.

I parameterize productivity of capital and labor as $\psi_i = \psi$ and $\gamma_i = 1/(1-i) - 1$. This ensures that γ/ψ converges to 0 and diverges as i goes to 0 and 1. Because there is only one type of capital in the model, which is a substitute for labor, I force all firms to use some capital. I choose ψ to match the wage premium of non-routine workers $(w_N/w_R - 1)$. Non-routine share parameter $1-\alpha$ chosen to match labor share of income. Humlum (2019) estimates elasticity of substitution parameter ρ as -1.

Following Eeckhout & Sepahsalari (2021), I parameterized matching function as

$$p(\theta) = (1 + \theta^{\omega})^{-1/\omega}.$$

Parameter	Target	Model	Data	Source
b	Replacement Rate	0.49	0.5	Department of Labor (2021)
$\underline{z}$	Employment Rate	0.94	0.94	FRED
λ_z	Sales Gini	0.54	0.85	Compustat
ω	Elasticity of $p(\theta)$	0.48	0.3-0.5	Pissarides (2009) & Shimer (2005)
α	Labor Share	0.67	0.67	BLS
λ	GS/GDP	0.29	0.25	OECD
ψ	Wage Premium	0.19	0.2	Guerreiro et al. (2021)

I choose ω to match the elasticity of the matching function with respect to θ to be around $0.3 - 0.5$, as empirically estimated by Pissarides (2009) and Shimer (2005). The vacancy posting cost function is assumed to be $v(\nu) = \nu^2$. The estimation of the convexity of the labor cost function is higher than three (Merz & Yashiv, 2007; Coşar et al., 2016), here I take a more conservative value of 2[2]. Observe that firm size depends on the convexity of vacancy cost because the production function is constant returns to scale. As the convexity of vacancy cost decreases, the profit share of firms goes down. In the extreme case, if it is linear, then only the top productive firm becomes active, and it serves all of the markets. However, because it operates constant returns to scale, its profit would be zero, and the model generates zero profit share.

Taxation scheme is parameterized as $T(w) = w - \lambda w^{1-\tau}$, following Heathcote et al. (2017). τ is a measure of progressivity. The higher it is, the more progressive income taxation is. Heathcote et al. (2017) find in the data that $\tau = 0.18$, hence I choose it as 0.18. λ is chosen to match the government spending to GDP ratio. The following table summarizes the calibration.

The model does not generate enough dispersion in sales. The Gini coefficient of sales distribution among Compustat firms was 0.85, whereas it is 0.54 in the model. This might be due to productivity distribution is not dispersed enough or the convexity of vacancy cost being too high so that the tail of the distribution does not generate relatively high economic

[2] Merz & Yashiv (2007) estimates the cost for hiring rate, not vacancy cost.

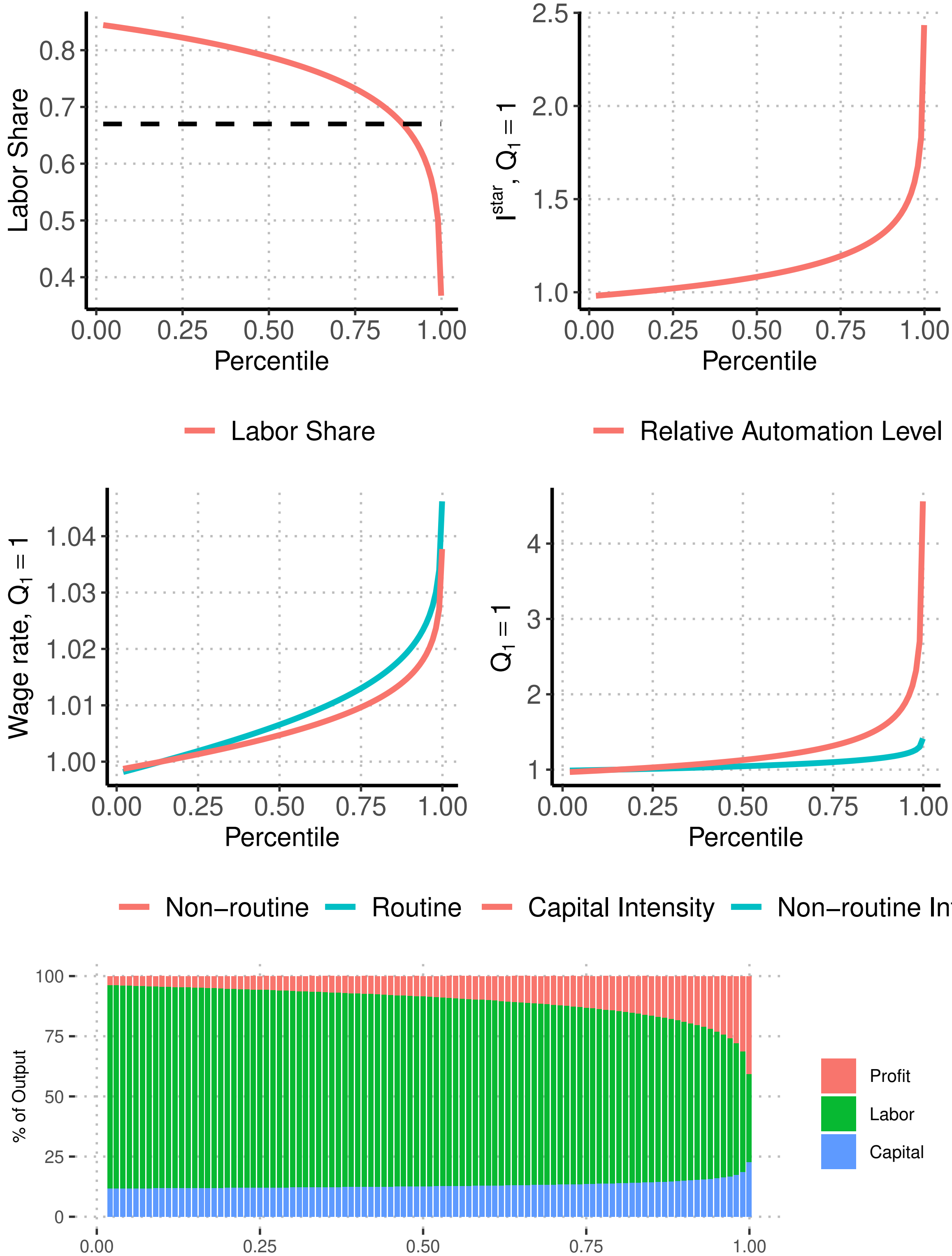
activity.

4.2 Initial Steady State

Figure 5 shows the selected statistics in the initial steady state. x-axis is the percentiles of firms according to the productivity distribution. The top left panel shows the labor share across firms, and as expected, labor share is decreasing with firm productivity. Up until the third 90^{th} percentile, the labor share is above the aggregate labor share (0.67). Because the economic activity is concentrated in the top firms, they derive the aggregate labor share down. There is a steep decrease in labor share at the top of the distribution. This is the result of the fact the fat tail of the Pareto distribution. The tail of the productivity distribution is more dispersed compared to the bottom of the distribution, and hence the decrease in the labor share becomes more pronounced within the top 10%.

The bottom panel of figure 5 shows the distribution of factor income. The bottom part (blue bar) is the capital share, the middle part (green bar) is the labor share, and the top part (red bar) is the profit share. As can be seen from the figure, both profit share and capital share are increasing with productivity. Because top firms use more capital-intensive technology, they replace labor with capital. Therefore capital share is increasing. Some of the labor shares are also captured by the firm directly in terms of profit. The difference in technological choice is clear from the top right and middle right panels. Former plots the automation decision, $I^{\star}$ relative to the first quantile. It shows that firms at the top percentile automate 2.5 times more compared to firms in the first quantile. Latter shows the capital intensity and non-routine intensity. Capital intensity is defined as capital to employment, and non-routine intensity is defined as non-routine employment to routine employment, N/R. The plot shows that capital intensity is higher for top firms, but the dispersion in non-routine intensity is not as high as capital intensity. As firms automate more, their demand for non-routine workers increases; however elasticity of non-routine labor demand with respect to automation decisions is not high, so the non-routine intensity is not steep.

Figure 5: Equilibrium Outcome in the Initial Steady State



Lastly, the model generates residual wage inequality (Burdett & Mortensen, 1998; Kaas & Kircher, 2015; Acemoglu & Shimer, 2000). The middle left panel shows the residual wage dispersion. The convexity of vacancy cost and productivity distribution generates wage dispersion in this model. It is known that models with search friction do not generate high wage dispersion as seen in the data (Hornstein et al., 2011). Because of the endogenous automation decision in the model, the wage dispersion is further suppressed. Instead of offering higher wages, top firms now automate more, which leads to a shrink in wage dispersion. It shows that the wage difference between workers employed at top firms and the bottom quantile is 4%. Doms et al. (1997) show that the wage difference between the top quantile and bottom quantile (in terms of technology adoption) is 8% for production workers after controlling for worker characteristics. This suggests that the model generates half of the observed wage dispersion.

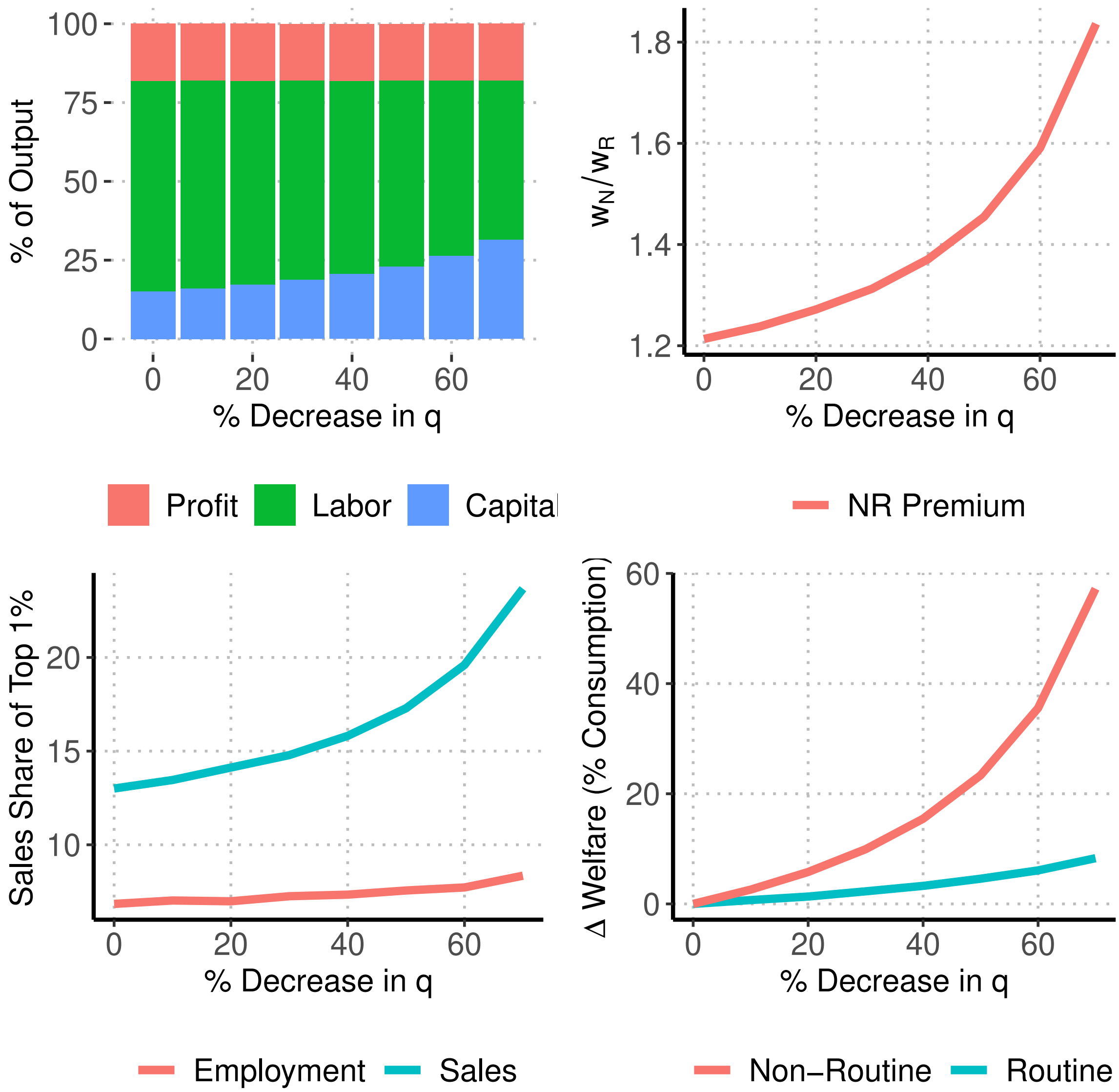
4.3 Impact of a Decrease in Price of Capital

In this section, I analyze the impact of a decrease in the price of capital. In the data, the decrease in investment price from the 1980s to the 2010s is 45%. However, I provide simulation results for a different levels of price decreases, ranging from 10% to 70%.

4.3.1 Aggregate Level Impact

Figure 6 shows the aggregate impact of a decrease in the price of capital q. Each plot has the same x-axis, a percentage decrease in the price of capital. The top left panel shows the change in the distribution of factor income. As expected, a decrease in the price of capital decreases the labor share of income. After a 45% decrease in the price of capital, labor share becomes 60%, whereas it is 62% in the data. An interesting implication of this model is that the profit share remains the same for a different level of price of capital. In other words, all of the decreases in labor share goes to the capital. This is in contrast to the finding of Barkai (2020). He shows that the decrease in the labor share is offset by the increase in

Figure 6: Aggregate Impact of a Decrease in the Price of Capital



the profit share. The model generates stable aggregate profit share even though the market concentration is increasing, as can be seen from the bottom left panel, which shows the share of the top 1% of firms in sales and employment. This implies that the profit share is also decreasing at the firm level so that reallocation does not lead to an increase in the profit share at the aggregate level. The reason for this is that capital is paid marginal product, whereas, due to search friction, labor is paid less than marginal product of labor. Therefore, the source of the profit is the difference between marginal product labor and the wage rate. Hence, moving technology from labor-intensive to capital-intensive leads to a reduction in the profit share. To be clear, the level of profit is rising, only the share in total income is decreasing.

The bottom left panel of figure 6 also shows the change in sales and employment concentration. The rise in the sales concentration is significant, whereas there is a slight increase in employment concentration. For a 45% decrease in price, sales concentration rises by 3.5 percentage points, and employment concentration increases by 0.5 percentage points. The observation that an increase in sales concentration is higher than employment concentration is in line with the finding of Leduc & Liu (2022), who estimate that the impact of robot per labor has a more pronounced impact on sales concentration than employment concentration. Notice that the composition of firms within percentiles is changing because of the exit of low-productive firms. As the wage rate increases, the profit of the lowest productive firm becomes negative, and, hence, it exits the market. Overall, the economy experiences higher firm productivity.

The graph on the top right panel of figure 6 plots the change in the non-routine wage premium, w_N/w_R. The decrease in the price of capital leads to the substitution of routine with capital, whereas an increase in non-routine because it complements capital. Therefore, the non-routine wage premium increases. For a 45% decrease in the price of capital, non-routine wage premium increases from 20% to 40%. In the date this wage premium increase to 35% (Guerreiro et al., 2021).

The graph on the bottom right panel of figure 6 shows the change in the welfare of routine and non-routine unemployed workers, only considering the wage income. In other words, it does not take into account the capital and profit income for non-routine workers. Not surprisingly, the main beneficiary of technological development is non-routine workers, and welfare impacts are large. For a 45% decrease in the price of capital, the welfare gain for the non-routine unemployed worker is a 20% increase in lifetime consumption. The same figure is 8% for routine unemployed. Routine workers are also gaining because employment is shifted to more productive firms. Even though there is no large increase in the employment share of the top 1%, reallocation generates this result. Moreover, routine workers are assigned to more productive tasks.

4.3.2 Firm Level Impact

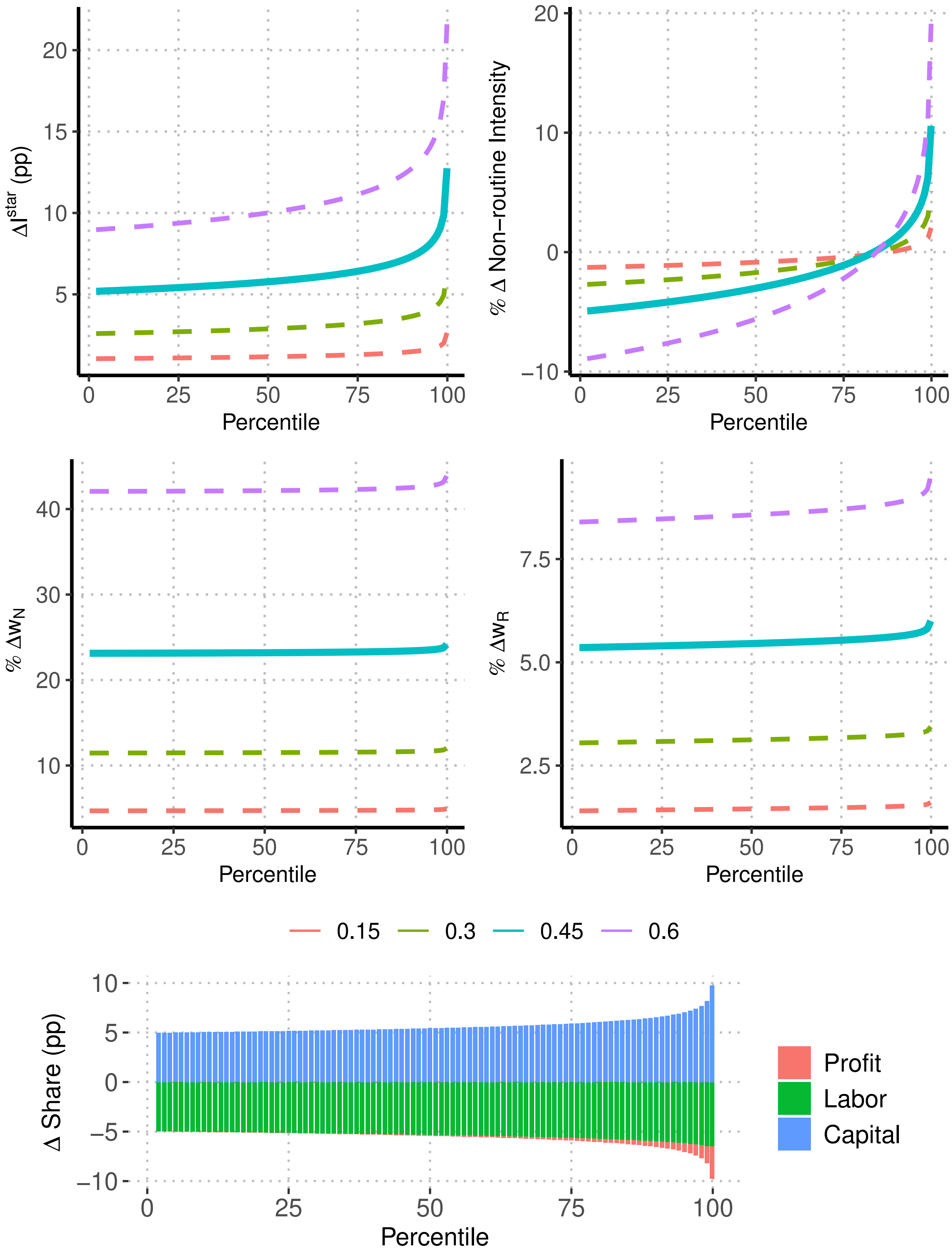
Now I turn to firm-level analysis. Figure 7 shows the change in selected statistics by percentile of the active firm productivity distribution. I show the results of simulations for different levels of price changes and emphasize the price decrease of 45% by the solid line, and other price changes are dashed lines. For the change in the factor income shares, I only show for 45% decrease.

In proposition 3, I show that a decrease in price leads to an increase in automation decisions. Furthermore, I showed that the change in wage rate is ambiguous. Top right panel shows that the second derivative of automation decision, $\frac{\partial I^{\star}(z;r)}{\partial r \partial z}$ is positive. In other words, a firm with high productivity adopts more technologies than firms with low productivity after a decline in the price of capital. Here, I plot for percentiles, and as I discussed above, the composition might be changing. However, this result is true if I were to plot the level of productivity on the x axis. This implies that top firms benefit from the advancement of automation technology the most. To put this number into perspective, Zolas et al. (2021) states that firms at the top 1% of the employment distribution are 1.7 times more likely to adopt new automation technologies than firms in the 50th and 75th percentile. This number is 2 in the model after a 45% decrease.

The second graph of figure 7 shows the change in non-routine intensity. It shows that firm at the top quartile increases their demand for non-routine workers more than demand for routine workers, whereas the opposite is true for the firms below the top quartile. The gradient becomes steeper as the price of capital goods decreases more.

The middle two graphs of figure 7 show the percentage change in routine and non-routine workers. As discussed above, because labor is employed at more productive firms and they are assigned to more productive tasks, it increases the wage of routine workers.

Figure 7: Firm Level Impact of a Decrease in the Price of Capital



However, even though the level of the change in the price of capital matters for the wage rate, firm productivity does not. Each curve is flat, with a small uptick at the top. Overall, inequality is increasing due to the difference between routine and non-routine. However, its contribution to residual inequality is minimal.

The bottom panel of figure 7 shows the change in the distribution of factor income. Here, I only plot this for a 45% price decrease. As can be seen from the graph, for the first three quartiles, the decrease in labor share is offset by an increase in capital share. For the firms in the top quartile, profit share is decreasing. As I discussed above, this is due to the assumption that cost of capital is linear, hence it is paid marginal productivity. Hence, as production become more capital intensive, profit share goes down. Aggregate profit share increases due to reallocation to large firms, but it decreases because their profit share reduced. Overall, these two effect cancel out and leads to a stable profit share in the aggregate level, as seen in figure 6. Even though the profit share is decreasing, profit level is increasing with productivity of firm.

Overall, figure 7 illustrates that firms at the top quartile benefited most from the improvement in technology, and the benefit is convexly increasing within the top quartile.

5 Capital Taxation

In this section, I analyze the impact of capital taxation. As discussed in 2.5, the market equilibrium is inefficient and can be recovered with regressive taxation, as in Golosov et al. (2013). However, in reality, progressive taxation is more prevalent, probably thanks to public support for progressive taxation (Ballard-Rosa et al., 2017). If the government cannot use progressive taxation, it can use other tools to reduce the inefficiency of the economy.

Recall that the main inefficiency is due to unemployment risk. Because there is no market for unemployment insurance, it leads to less than optimal risk-taking. In other words, individuals are not searching for high-paying jobs enough. This implies that the wage rate is less than the socially optimal level, incentivizing firms to use more labor-intensive technologies. On the other hand, because workers do not search for high-paying jobs, this hurts highly productive firms more than lowly productive firms. This is because highly productive firms cannot offer high wages to increase their vacancy-filling rate; hence they automate more than the socially optimal level to economize on vacancy posting costs.

This leads to inefficient use of capital: small firms use capital less than optimal, and large firms use more than optimal. By using capital taxation or subsidy, the government can impact the capital intensity, hence the welfare. The sigh of this tax depends on which part dominates. Quantitatively, it turns out that small firms that are not using enough capital are more dominant.

To quantitatively figure out the optimal capital taxation, I recalibrate my model to the 2010s economy. I assume that government needs to pay the government spending implied by calibration, and any additional tax revenue is used as a transfer to unemployed individuals. Initially, I keep income tax and profit tax at the calibrated value and change the capital tax level to find the rate that maximizes newborn unemployed. Specifically, I consider the following problem:

$$\max_{\tau_K} \mu_R U_R + \mu_N U_N.$$

I find that a 15% subsidy to capital is optimal. Even though it decreases the routine unemployed by 2.5% lifetime consumption, it generates a 13% increase in lifetime consumption of non-routine unemployed. This figure does not take into account the increase in capital income and profits. If we take that into account, the figure would be more significant.

6 Conclusion

This paper analyzes the impact of a decrease in the relative price of investment goods on a firm's technology decision and distribution of factor income. I consider a model with search friction leading to technological adoption heterogeneity. In the model, highly productive firms automate more, and because of the convex cost of vacancy posting, they face a higher marginal cost of labor, hence wanting to automate more. A decrease in the price of capital generates a heterogeneous impact across firms. Large firms automate more relative to small firms. Hence they capture a larger share of the market. The reallocation of economic activity to large firms leads to a decline in labor share and an increase in capital share.

The inefficiency due to unemployment risk creates a role for the government. Without regressive taxation, the government can tax capital (or subsidize it) to distort firms' technology choices. Here I consider an optimal tax rate given the calibrated income tax schedule. The obvious step is to take into account changes in income taxation too.

One important avenue for future research is firm dynamics. I consider a steady-state model without productivity shocks. However, the main channel is also crucial for growing firms. The model implies that young firms automate more along their transition to optimal levels, and as they mature, they tend to decrease their automation. This is an interesting and open question.

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Appendix

A Proofs

A.1 Proof of Lemma 1

Proof. *By assumption, $\theta \to \infty$ implies that $p(\theta) \to 1$. Hence $w(\theta) \to u^{-1}[S + u(b)]$. Because S and b are finite, then right hand side it finite.*

By assumption, p converges to 1, hence its derivative converges to 0. Because limit of $w(\theta)$ is finite, $S/(p(\theta)u'(w(\theta))$ converges to a finite number. Hence, $w'(\theta)$ converges to 0.

■

A.2 Proof of Lemma 2

Proof. *Consider equation*

$$\frac{r\gamma_I}{\psi_I} = w(\theta) + w'(\theta)\frac{q(\theta)}{q'(\theta)}$$

By assumption, the left hand side is increasing in I, and right hand side is decreasing in θ. Hence, an increase in I decreases θ. ■

A.3 Proof of Lemma 3

Proof. *Solving for L in equation (5):*

$$L^{1-\alpha} = z\alpha\left[\phi(I) + \eta(I)\right]^{\frac{\alpha}{\rho}-1}\left(\frac{\eta(I)}{\gamma_I}\right)^{1-\alpha}\frac{\psi_I}{r}. \tag{7}$$

Now, I show that L is decreasing in I.

$$\left(\frac{\alpha-\rho}{\rho}\right)(\eta+\phi)^{\alpha/\rho-2}(\eta'+\phi')\psi\left(\frac{\eta}{\gamma}\right)^{1-\alpha}+(\eta+\phi)^{\alpha/\rho-1}\psi'\left(\frac{\eta}{\gamma}\right)^{1-\alpha}$$
$$+(\eta+\phi)^{\alpha/\rho-1}\psi\left(\frac{\eta}{\gamma}\right)^{-\alpha}(1-\alpha)\left(\frac{\eta'}{\gamma}-\frac{\eta\gamma'}{\gamma^2}\right)$$

This also has the same sign with:

$$\left(\frac{\alpha-\rho}{\rho}\right)(\eta'+\phi')\left(\frac{\eta}{\gamma}\right)+(\eta+\phi)\frac{\psi'}{\psi}\left(\frac{\eta}{\gamma}\right)+(\eta+\phi)^{\alpha/\rho-1}(1-\alpha)\left(\frac{\eta'}{\gamma}-\frac{\eta\gamma'}{\gamma^2}\right)$$
$$=-\frac{\alpha-\rho}{1-\rho}\frac{\eta}{\gamma}\left(\eta\frac{\gamma'}{\gamma}+\phi\frac{\psi'}{\psi}\right)+(\eta+\phi)\frac{\eta}{\gamma}\frac{\psi'}{\psi}+(\eta+\phi)(1-\alpha)\left(-\frac{1}{\gamma}-\frac{\rho}{1-\rho}\frac{\eta}{\gamma}\frac{\gamma'}{\gamma}-\frac{\eta}{\gamma}\frac{\gamma'}{\gamma}\right)$$
$$<-\frac{\alpha-\rho}{1-\rho}\frac{\eta}{\gamma}\left(\eta\frac{\gamma'}{\gamma}+\phi\frac{\psi'}{\psi}\right)+(\eta+\phi)\frac{\eta}{\gamma}\frac{\psi'}{\psi}\frac{\alpha-\rho}{1-\rho}<0$$

Last inequality follows from the assumption that γ/ψ is increasing and $\gamma, \psi, 1-\alpha, \gamma$ are all positive. Hence, L is decreasing in I.

Now, because $L(I)$ is decreasing, then the left hand side of the following equation is decreasing with I.

$$v'\left(\frac{L}{q(\theta)}\right)\frac{1}{q(\theta)}=w'(\theta)\frac{q(\theta)}{q'(\theta)} \tag{8}$$

Observe that right hand side is decreasing in θ, and right hand side is increasing with θ. So it implies that an increase in I leads to an increase in θ. An increase in I decreases LHS, if θ decreases, LHS decreases further while RHS increases. Hence θ must go up to bring this equation to equality.

■

A.4 Proof of Proposition 1

We want to find $I^\star$ such that $\theta_1(I^\star) = \theta_2(I^\star)$. To show that solution exists, I need to show that end points are on the opposite side of each other. Because θ_1 is strictly decreasing and θ_2 strictly increasing, $\theta_1(0) < \theta(2)$ and $\theta_1(1) > \theta_2(1)$ implies that they must intersect. Strict monotonicity implies that they can intersect only once.

First, consider as $I \to 1$. In this case, by assumption, left hand side of equation (6) is positive. This implies that $\theta_1(I)$ converges to a finite number as $I \to 1$. If relative productivity of labor diverges, then $\theta_1(I)$ converges to 0. Recall that $L(I; z)$ is decreasing in I, and it goes to 0 as $I \to 1$. This implies that left hand side of equation (8) converges to 0. Observe that right hand side goes to 0 as $\theta \to \infty$. This implies that $\theta_2(I) \to \infty$ as $I \to 1$. In other words, as I converges to 1, $\theta_1(I) < \theta_2(I)$.

Now consider $I \to 0$. The left hand side of equation(6) converges to $u^{-1}[S + u(b)]$ by lemma 1. If relative productivity γ_I/ψ_I converges to a number strictly below limit of the wage, then $\theta_1(0)$ exists. Otherwise, it does not exists and there exists minimum automation level I_{min} such that $\theta_1(I) \to \infty$ as $I \to I_{min}$. If relative productivity converges to a positive number, then labor demand converges to a positive number finite number. Hence $\theta_2(I)$ converges to a positive finite number. If relative productivity converges to 0, then labor demand diverges, and hence $\theta_2(I)$ goes to 0. In either case, limit of θ_2 exists.

This implies that if the lower limit of relative productivity is below limit of the wage rate, then there exists $I^\star > I_{min}$ such that $\theta_1(I^\star) = \theta_2(I^\star)$. Otherwise, there are two cases. One, if $\theta_2(0) > \theta(0)$, then $I^\star = 0$. Two, if $\theta_1(0) > \theta_2(0)$, then $I^\star > 0$.

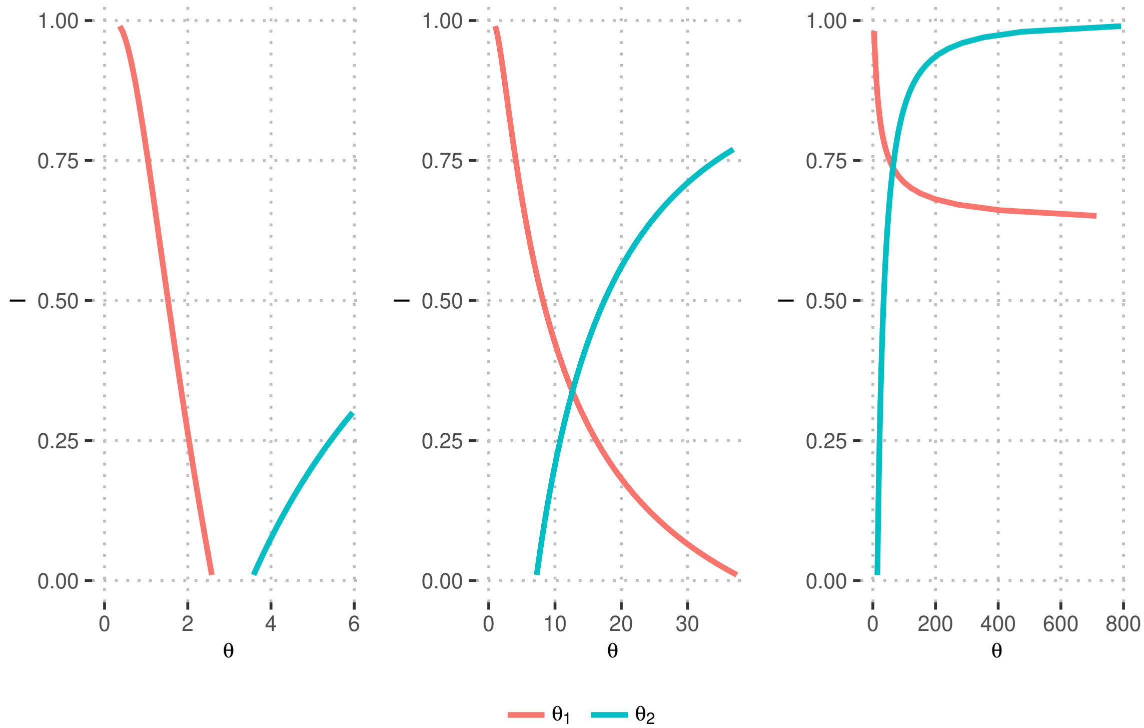
In summary:

1. As $I \to 1$, $\theta_1(I) < \theta_2(I)$, because $\theta_1(I) < \infty$ and $\theta_1 \to \infty$.

2. As $I \to 0$:

   (a) if $\lim \gamma/\psi < u^{-1}[S + u(b)]$: $\theta_2(0) < \theta_1(I_{min}) \to \infty$, where I_{min} satisfies $\gamma/\psi =$

Figure 8: Solution to the Firm's Problem



$u^{-1}[S + u(b)]$,

(b) if $\lim \gamma/\psi \geq u^{-1}[S + u(b)]$: then $\theta_1(0) > 0$ and finite.

i. if $\theta_1(0) > \theta_2(0)$: $I^\star > 0$.

ii. if $\theta_1(0) \leq \theta_2(0)$: $I^\star = 0$.

A.5 Proof of Proposition 2

Proof. *Observe that $\theta_1(I)$ does not depend on z. z only impacts labor demand through equation* (5)*. An increase in z increases labor demand for a given automation level. This leads to a decrease in θ in equation* (2)*.* ■

A.6 Proof of Proposition 3

Proof. *A decrease in the price of capital r makes automation cheaper. From equation (6), wage should go down, hence $\theta_1(I)$ increases.*

From equation (7), a decrease in r increases L. From (2), this leads to a lower θ. Hence $\theta_2(I)$ decreases. To bring back the equilibrium, I must increase. Impact on θ is ambiguous.

■

A.7 Proof of Proposition 4

The problem is similar to Golosov et al. (2013), with inclusion of large firms that chooses production technology. Similar results follow this paper.

Proof. *Let subscript e denotes the efficient outcome and $\star$ denotes the market outcome. Objective is to find tax schedule $\tau(\theta)$ that recovers the efficient allocation: $\theta^e(z) = \theta^\star(z)$ and $I^e(z) =$. Efficiency requires that unemployed individuals should consume $I^\star(z)$. Efficiency requires that consumption of unemployed must be $b + T = c_u^e$, where T is the transfer to unemployed, and consumption of employed is $(1 - \tau(\theta))w(\theta^\star(z)) = c_z^e$. If these are true, then $L^e(z) = L^\star(z)$ since there is no additional distortion for firm's problem. This also implies tat goods and labor markets are cleared in market equilibrium.*

$\{\theta^e, I^e\}$ solves optimality condition for efficient outcome, and $\{\theta^\star, I^\star\}$ solves optimality conditions for market outcome.

$$\begin{aligned}
\frac{v'(L)}{q(\theta)} =& \frac{\zeta(\theta}{p(\theta)}\left[\frac{S}{u'(c_{z(\theta)})}+\lambda\right],\\
\frac{v'(L)}{q(\theta)} =& w'(\theta)\frac{q(\theta)}{q'(\theta)}\\
\implies w'(\theta)\frac{q(\theta)}{q'(\theta)} =& \frac{\zeta(\theta}{p(\theta)}\left[\frac{S}{u'(c_{z(\theta)})}+\lambda\right].
\end{aligned}$$

Similarly:

$$\frac{\gamma_I r}{\psi_I} = c_{z(\theta)} + b - c_u + \frac{\zeta(\theta)}{p(\theta)}\frac{S}{u'(c_\theta)} + \frac{\lambda}{p(\theta)}[\zeta(\theta)+1], \tag{9}$$

$$\frac{\gamma_I r}{\psi_I} = w(\theta) + w'(\theta)\frac{q(\theta)}{q'(\theta)} \tag{10}$$

$$\implies w'(\theta)\frac{q(\theta)}{q'(\theta)} = c_{z(\theta)} + b - c_u + \frac{\zeta(\theta)}{p(\theta)}\frac{S}{u'(c_\theta)} + \frac{\lambda}{p(\theta)}[\zeta(\theta)+1]. \tag{11}$$

Combining these two results implies that:

$$w(\theta) = c_{z(\theta)} + b - c_u + \frac{\lambda}{p(\theta)}.$$

Because $(1-\tau(\theta))w(\theta) = c_\theta$*:*

$$1-\tau(\theta) = \frac{c_{z(\theta)}}{c_{z(\theta)} - c_u + b + \lambda/p(\theta)}.$$

■